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REPORT NO. 1330

TAKE-OFF GROUND ROLL OF A JET DRIVEN AND  
PROPELLER DRIVEN AIRPLANE EQUIPPED WITH  
THE INTEGRAL AND THE INDEPENDENT FORCED  
CIRCULATION SYSTEM

**Cessna Aircraft Company**  
**Wichita, Kansas**

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**Cessna**  
**Aircraft Company**  
Wichita, Kansas

Aircraft Division - Research Section

MODEL

REPORT NO. 1320

TAKE-OFF GROUND ROLL OF A JET DRIVEN AND  
PROPELLER DRIVEN AIRPLANE EQUIPPED WITH  
THE INTEGRAL AND THE INDEPENDENT FORCED

CIRCULATION SYSTEM

REPORT DATE: March 22, 1954

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FOREWORD

This report was prepared by the Research  
Section, Aircraft Division of Cessna Aircraft  
Company, Wichita, Kansas under the contract  
NOMR 856(00) with the Office of Naval Research,  
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LIST OF SYMBOLS

$a$	Horizontal acceleration, ft/sec. <sup>2</sup>
$c$	Wing chord at 75% propeller radius, ft.
$\bar{c}$	Wing mean aerodynamic chord, ft.
$g$	Gravity acceleration
$w$	Total weight of the forced circulation system.
$C_L$	Lift coefficient
$C_{L_t}$	Lift coefficient at take-off (not adjusted for the effect of slipstream)
$C_D$	Drag coefficient
$C_{D_p}$	Parasite drag coefficient
$D$	Drag, lbs.
$D$	Propeller diameter, ft.
FC	Abbreviation for forced circulation, which is boundary layer control at high lift by sucking and/or blowing air through slots ahead of deflected trailing edge flaps and drooped ailerons.
$L$	Lift, lbs.
HP	Horsepower of the main engine
hp	Horsepower required by FC system
$\lambda$	Thrust reduction factor
$S$	Wing area, sq.ft.
$S_g$	Take-Off ground roll, ft.
$T_0$	Static thrust, lbs.
$T$	Thrust at any velocity, lbs.
$V$	Velocity, ft./sec.
$V_t$	Take-off velocity, ft./sec.
$W$	Gross weight of the airplane, lbs.

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LIST OF SYMBOLS

$\Delta C_L$   
 $\frac{\Delta C_L}{\Delta hp}$  Increase of lift coefficient per horsepower absorbed by FC system

$\frac{\Delta w}{\Delta hp}$  Increase of weight per horsepower absorbed by FC system

$\mu$  Coefficient of ground friction

$\rho$  Mass density of air

$\epsilon$  Downwash angle, radians

Primed symbols indicate that FC system is in operation.

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MODEL \_\_\_\_\_SUMMARY

Equations for take-off ground roll in terms of three parameters:

$$\frac{hp}{HP}, \frac{\Delta w}{\Delta hp}, \text{ and } \frac{\Delta C_L}{\Delta hp}$$

are derived for an airplane equipped with the integral and the independent forced circulation system. In the case of a propeller driven airplane, the effect of slipstream upon take-off lift coefficient is taken into consideration.

The numerical example illustrating application of the method to a hypothetical small jet powered and propeller powered aircraft is presented.

Results indicate that the independent system is more effective in reduction of take-off ground roll than the integral system.

INTRODUCTION

The problem of ground run during take-off was analyzed and solved many years ago by Diehl and later elaborated upon by others. The basic equation of motion is well understood and the results are usually given in terms of the net accelerating force and the velocity of take-off.

The application of forced circulation (abbreviated from now on as FC) introduces several new factors which affect the length of take-off ground run and at the same time, complicates analysis of these factors with respect to their relative importance.

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This report is a review of the problem as it is affected by the application of FC and an attempt to analyze the effect of the three major parameters

$$\frac{hp}{EP}, \frac{\Delta w}{\Delta hp}, \text{ and } \frac{\Delta C_L}{\Delta hp}$$

upon take-off performance.

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### I. JET DRIVEN AIRCRAFT (SLIPSTREAM IS ABSENT)

Application of forced circulation presents two cases:

(A) Integral FC System where the main engine of the airplane supplies motive power to pump air.

(B) Independent FC System where power to pump air is obtained from other sources than the main engine. Equations for take-off ground run with FC system in operation and the criterion for the optimum division of power between the main engine and FC system will be developed for each case.

#### A. Integral FC System

It can be shown (See Appendix) that ground run during take-off can be approximated by:

$$S_g = \frac{W}{\rho g R} \left[ \left( \frac{T_0}{W} - \mu \right) C_{L_t} - R \right] \quad (9)$$

Since a portion of the main engine power is diverted to operate the FC system, the reduced static thrust can be expressed as:

$$T_0' = T_0 \left( 1 - \frac{hp}{HP} \right) \quad (16)$$

Similarly, the increased take-off lift coefficient can be written as:

$$C_{L_t}' = C_{L_t} + \left( \frac{\Delta C_L}{\Delta hp} \right) hp \quad (17)$$

Finally, the increased gross weight due to the installation of the system can be accounted for as:

$$W' = W + \left( \frac{\Delta v}{\Delta hp} \right) hp \quad (18)$$

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Substituting (16), (17), and (18) into (9) we obtain:

$$s_g' = \frac{\frac{1}{\rho g S} \left[ W + \left( \frac{\Delta w}{\Delta hp} \right) hp \right]}{\left[ \frac{T_o (1 - \frac{hp}{HP})}{W + \left( \frac{\Delta w}{\Delta hp} \right) hp} - \mu \right] \left[ C_{L_t} + \left( \frac{\Delta C_L}{\Delta hp} \right) hp \right] - R'} \quad (19)$$

Equation (9) and (19) furnish direct comparison of the take-off ground run of an airplane without and with FC when a certain horsepower is diverted from the main engine to operate the FC system.

If the optimum division of power,  $\left( \frac{hp}{HP} \right)_{opt.}$  is sought, equation (19) can be written in the following form:

$$s_g' = \frac{\frac{HP}{\rho g S} \left[ \frac{W}{HP} + \left( \frac{\Delta w}{\Delta hp} \right) x \right]}{\left[ \frac{T_o (1 - x)}{\frac{W}{HP} + \left( \frac{\Delta w}{\Delta hp} \right) x} - \mu HP \right] \left[ \frac{C_{L_t}}{HP} + \left( \frac{\Delta C_L}{\Delta hp} \right) x \right] - R'} \quad (19a)$$

where  $x = \frac{hp}{HP}$

Differentiating equation (19a) with respect to  $x$ , equating to zero and solving for  $x$  we obtain:

$$\left( \frac{hp}{HP} \right)_{opt.} = \frac{\frac{\Delta C_L}{\Delta w} \cdot \frac{W}{HP} (T_o - \mu W) - \frac{C_{L_t}}{HP} (2T_o - \mu W) - \frac{W}{HP} \left( T_o \frac{C_{L_t} \Delta hp}{HP \Delta w} - R' \right)}{\frac{\Delta C_L}{\Delta hp} (T_o + \mu W) - \frac{\Delta w}{\Delta hp} \left( \mu C_{L_t} + R' \right) + T_o \left( \frac{2W}{HP} \cdot \frac{\Delta C_L}{\Delta w} - \frac{C_{L_t}}{HP} \right)} \quad (20)$$

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where  $\frac{\Delta C_L}{\Delta w} = \frac{\Delta C_L}{\Delta hp} \cdot \frac{\Delta hp}{\Delta w}$

The shortest take-off ground,  $S_g''$  min. can be calculated by substituting the numerical value of  $\left(\frac{hp}{HP}\right)_{opt.}$  into (19a).

### B. Independent System

Since in this case power to operate the system is not taken off the main engine equation (16) reduces to

$$T'_0 = T_0$$

Then equation (19) takes the following form:

$$S_g'' = \frac{\frac{1}{\rho g S} \left[ W + \left( \frac{\Delta w}{\Delta hp} \right) hp \right]}{\left[ \frac{T_0}{W + \left( \frac{\Delta w}{\Delta hp} \right) hp} - \mu \right] \left[ C_{L_{t_0}} + \left( \frac{\Delta C_L}{\Delta hp} \right) hp \right] - R''} \quad (21)$$

Equations (9) and (21) allow comparison of the take-off ground run of an airplane without and with FC when a given hp to operate FC system is supplied by an auxiliary power plant.

Again, if the optimum division of power is desired, equation (21) can be written as:

$$S_g'' = \frac{\frac{HP}{\rho g S} \left[ \frac{W}{HP} + \left( \frac{\Delta w}{\Delta hp} \right) x \right]}{\left[ \frac{T_0}{\frac{W}{HP} + \left( \frac{\Delta w}{\Delta hp} \right) x} - HP\mu \right] \left[ \frac{C_{L_t}}{HP} + \left( \frac{\Delta C_L}{\Delta hp} \right) x \right] - R''} \quad (21a)$$

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as, before, by minimizing equation (21a) we obtain:

$$\frac{\frac{\Delta C_L}{\Delta v} \cdot \frac{W}{HP} (T_0 - \mu W) - \frac{C_{L_t}}{HP} (2T_0 - \mu W) + \frac{W}{HP} R^*}{\frac{hp}{HP_{opt}}} \quad (22)$$

$$\frac{\Delta C_L}{\Delta hp} (T_0 + \mu W) - \frac{\Delta v}{\Delta hp} (R^* + \mu C_{L_t})$$

The shortest take-off -  $S_{g^* \min}$ . can be obtained by substituting numerical value of  $(\frac{hp}{HP_{opt}})$  into equation (21a). Equations (20) and (22) provide a convenient means to study the effect of the three major parameters

$$\frac{hp}{HP}, \frac{\Delta C_L}{\Delta hp} \quad \text{and} \quad \frac{\Delta v}{\Delta hp}$$

upon take-off ground roll.

## II. PROPELLER DRIVEN AIRCRAFT (SLIPSTREAM PRESENT)

### A. Integral System

Substitution of (16), (17) and (18) into (12) yields:

$$S_{g^*} = \frac{\frac{1}{\rho s} \left[ W + \left( \frac{\Delta v}{\Delta hp} \right) hp \right]}{\left[ \frac{T_0 \left( 1 - \frac{hp}{HP} \right)}{W + \left( \frac{\Delta v}{\Delta hp} \right) hp} - \mu \right] \left[ C_{L_t} + \Delta C_{L_S} + \left( \frac{\Delta C_L}{\Delta hp} \right) hp - R^* \right]} \quad (23)$$

where:

$$\Delta C_{L_S} = \frac{1}{2} \frac{\partial}{\partial} \left[ C_{L_t} + \left( \frac{\Delta C_L}{\Delta hp} \right) hp \right]^2 \left[ \frac{T_0 \left( 1 - \frac{hp}{HP} \right)}{W + \left( \frac{\Delta v}{\Delta hp} \right) hp} - \frac{R^*}{2s \left[ C_{L_t} + \left( \frac{\Delta C_L}{\Delta hp} \right) hp \right]} \right]$$

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Expressed in terms of  $x = \frac{hp}{HP}$ , equation (23) can be written as:

$$S'g = \frac{\frac{HP}{\rho g S} \left[ \frac{W}{HP} + \left( \frac{\Delta v}{\Delta hp} \right) x \right]}{\left[ \frac{T_0 (1-x)}{\frac{W}{HP} - \left( \frac{\Delta v}{\Delta hp} \right) x} - \mu HP \right] \left[ \frac{C_{L_t}}{HP} + \frac{\Delta C_{L_S}}{HP} + \left( \frac{\Delta C_L}{\Delta hp} \right) x \right] - R^1} \quad (23a)$$

where

$$\frac{\Delta C_{L_S}}{HP} = \frac{1}{2} \frac{C}{\rho} \left[ \frac{C_{L_t}}{HP} + \left( \frac{\Delta C_L}{\Delta hp} \right) x \right]^2 \left[ \frac{T_0 (1-x)}{\frac{W}{HP} + \left( \frac{\Delta v}{\Delta hp} \right) x} - \frac{K}{2S \left[ \frac{C_{L_t}}{HP} + \left( \frac{\Delta C_L}{\Delta hp} \right) x \right]} \right]$$

Differentiation of equation (23a) leads to a cumbersome expression which defeats the purpose of the analysis and therefore is not presented here. The optimum  $\frac{hp}{HP}$  and therefore the minimum  $S'g$  can be calculated by solving equation (23a) for several values of  $x$ .

#### B. Independent System

As before, in this case  $T'_0 = T_0$  and equation (23) and (23a) are still valid provided that  $T_0$  is substituted for  $T_0 (1-x)$ .

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Numerical Example

The following numerical constants were used in order to illustrate method of analysis:

$$W = 2,000 \text{ lbs.}$$

$$S = 175 \text{ sq.ft.}$$

$$HP = 200$$

$$T_0 = 1,000 \text{ lbs.}$$

$$\mu = .05$$

$$M = .6$$

$$R = .200 \text{ (without FC)}$$

$$R' = .150 \text{ (with FC)}$$

$$C_{L_t} = 1.5 \text{ (Case 1 - Jet Driven Aircraft)}$$

$$C_{L_t} = 1.5 \text{ (Case 2 - Propeller Driven Aircraft)}$$

$$\frac{\Delta v}{\Delta \text{hp}} = 3, 5, \text{ and } 7$$

$$\frac{\Delta C_L}{\Delta \text{hp}} = .03, .04 \text{ and } .05$$

The listed above constants were inserted in equations (19a), (21a), and (23a). The equations were solved for x ranging from 0 to 1.0 and results are presented in Figs. (2) and (3).

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CONCLUSIONS

The comparative charts (Fig. 2) and (Fig. 3) show that in both cases, with and without slipstream the independent FC system holds more promise of reduction in take-off ground roll than the integral system under the same conditions. This can be explained by the decrease of engine thrust due to absorption of power by FC system.

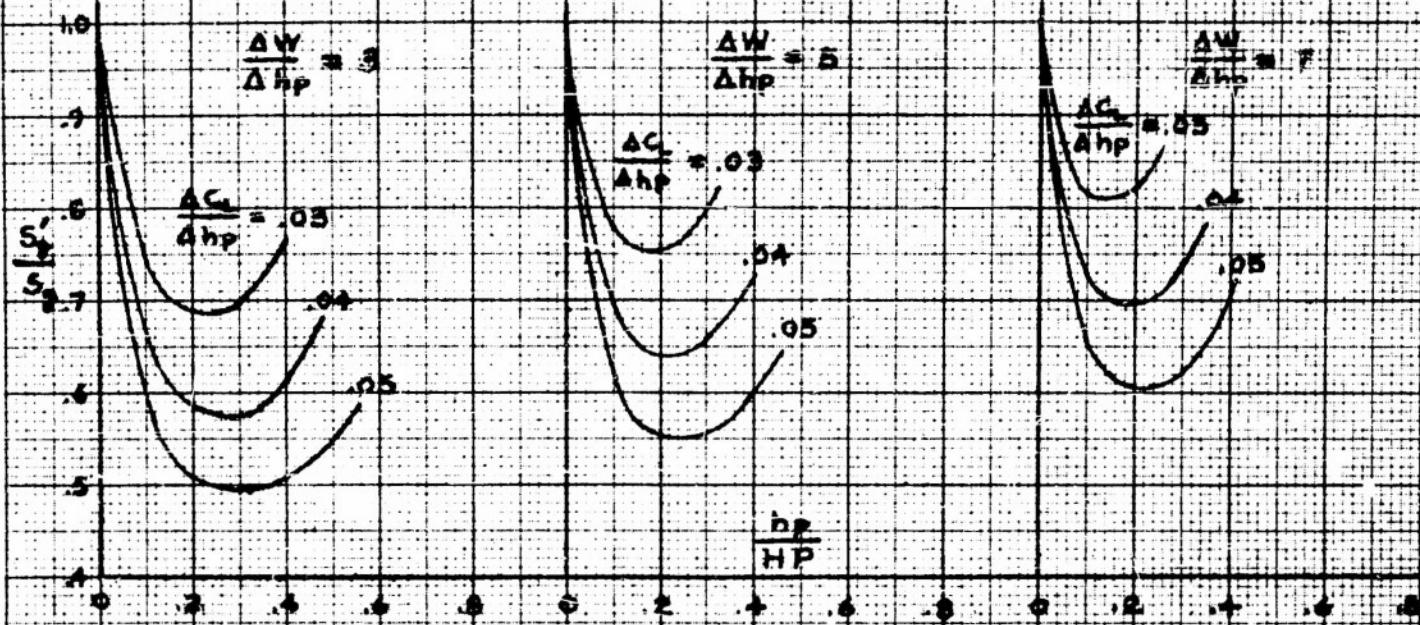
In the case of independent system, the maximum reductions in ground roll occur at much higher power ratios than for the integral systems.

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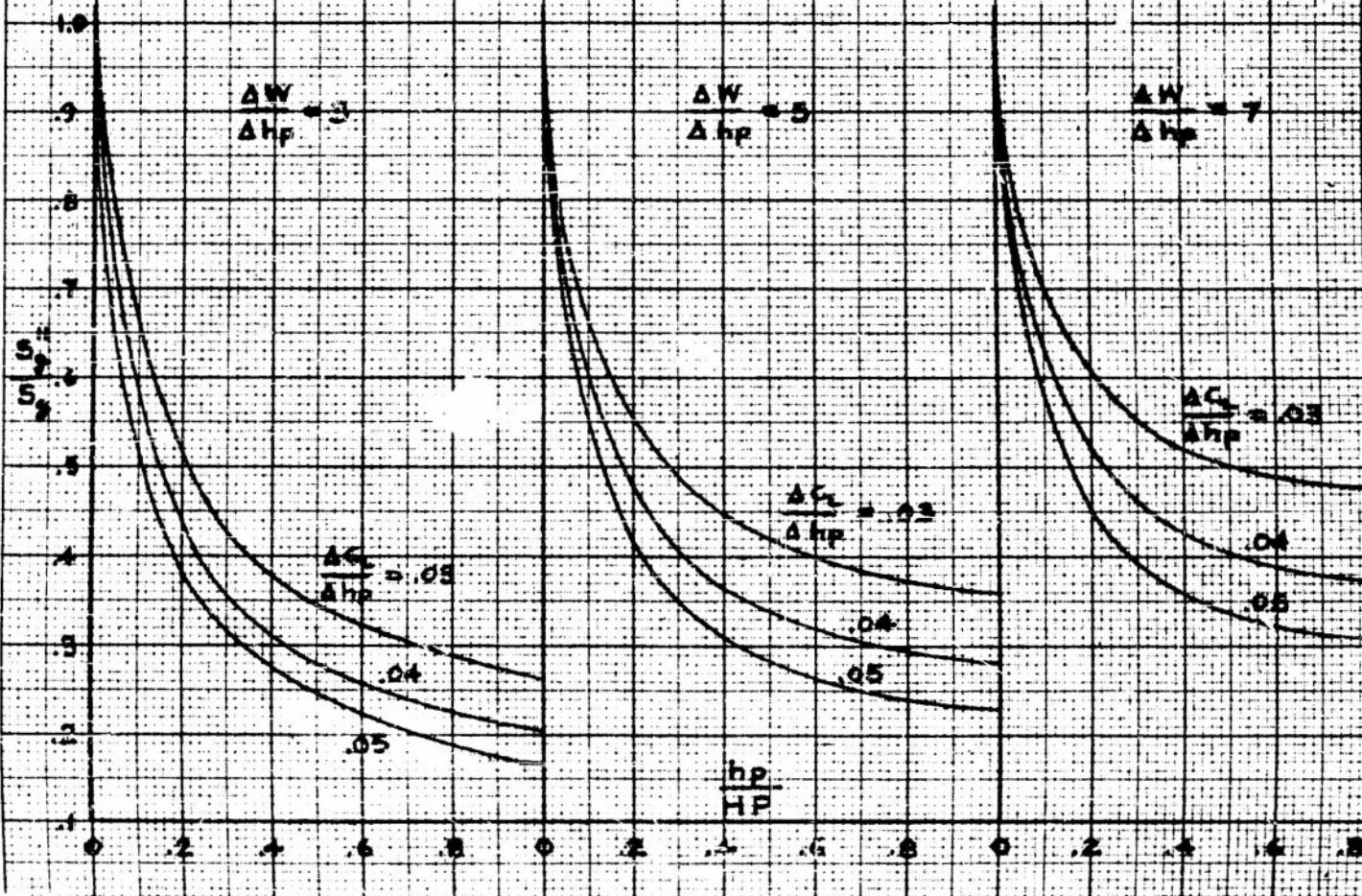
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## JET DRIVEN AIRCRAFT

## INTEGRAL SYSTEM

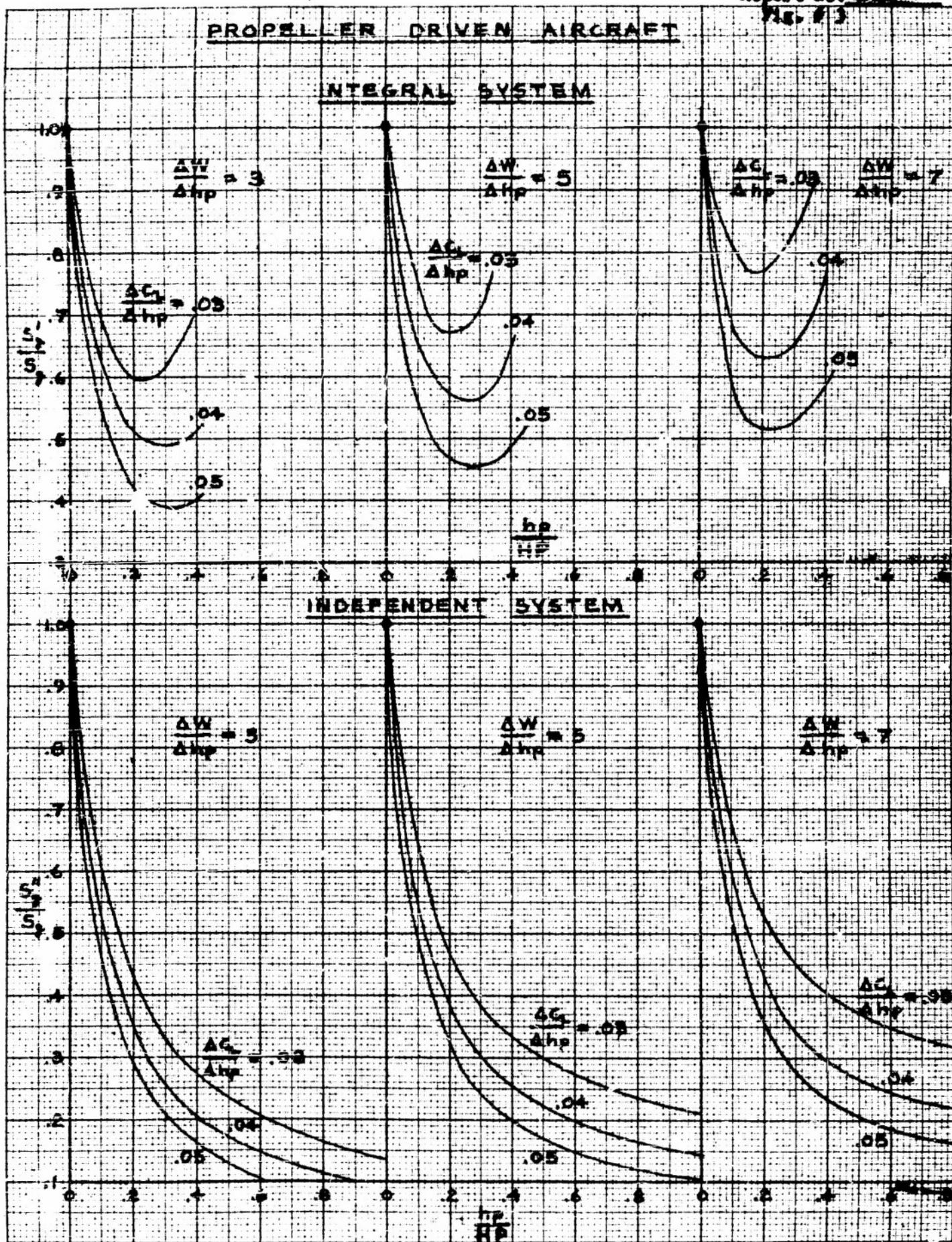


## INDEPENDENT SYSTEM



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APPENDIXEQUATIONS FOR TAKE-OFF GROUND ROLLAssuming that the decrease in static thrust is a function of  $V^2$ , or

$$T = T_0 - 1/2 \rho V^2 K \quad (1)$$

the ground run of an airplane during take-off can be calculated with sufficient accuracy by

$$S_g = V_t^2 / 2a \quad (2)$$

where horizontal acceleration

$$a = g/W [T - D - \mu (W - L)] \quad (3)$$

is computed at  $V = \sqrt{.5} V_t$ 

The numerical value of this constant is based on the fact verified by numerous flight tests that the reciprocal of the horizontal acceleration is very nearly a linear function of the velocity squared. (Ref. 2)

Substitution of (1) into (3) gives:

$$a = g [T_0/W - \mu - 1/2 \rho V^2 S/W (C_D - \mu C_L + K/S)] \quad (4)$$

At  $V = \sqrt{.5} V_t$  equation (4) can be written as:

$$a = g [T_0/W - \mu - .5/C_{L_t} (C_D - \mu C_L + K/S)] \quad (5)$$

Substitution of (5) into (2) yields:

$$S_g = \frac{W}{\rho g S [(T_0/W - \mu) C_{L_t} - .5 (C_D - \mu C_L + K/S)]} \quad (6)$$

For the given static thrust, gross weight, coefficient of ground friction and take-off lift coefficient, ground run is a minimum when the term

$$C_D - \mu C_L + \frac{K}{S}$$

vanishes. This term can be written as:

$$C_{D_p} + \beta C_L^2 - \mu C_L + K/S \quad (7)$$

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where

$$\beta = \frac{dC_D}{dC_L^2}$$

is a slope of the linearized polar of the airplane. Minimizing (7) with respect to  $C_L$  we obtain:

$$C_L = \frac{\mu}{2\beta}$$

Then, for minimum air and ground resistance term (7) becomes:

$$C_{D_p} = \frac{\mu^2}{4\beta} + \frac{K}{S} \quad (8)$$

Substituting (8) into (6) we obtain:

$$S_2 = \frac{W}{\rho g S \left[ \left( \frac{T_0}{W} - \mu \right) C_{L_t} - R \right]} \quad (9)$$

where

$$R = .5 (C_{D_p} - \frac{\mu^2}{4\beta} + \frac{K}{S})$$

The numerical value of  $K$  is obtained from the propeller characteristic curve and  $C_{D_p}$  and  $\beta$  are determined from the polar curve of the airplane in the take-off configuration, with and without F.O.

Up to this point the effect of propeller slipstream has not been taken into consideration implying that equation (9) is applicable only to jet powered aircraft.

The effect of slipstream upon take-off lift coefficient can be expressed as:

$$\Delta C_{L_S} = N T_e C_L - \frac{c}{S} \frac{D^2}{S} \quad (\text{Reference 12}) \quad (10)$$

where  $N$  is a constant, numerical value of which depends upon flap deflection and the number of propellers.

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$$\text{Since } T_0 = \frac{T}{\rho v^2 D^2}$$

$$T = T_0 - \frac{1}{2} \rho v^2 k$$

$$v = \sqrt{.5} v_t$$

equation (10) can be written as:

$$\Delta C_{L_s} = \frac{1}{2} C_{L_t}^2 \left( \frac{T_0}{W} - \frac{k}{20 C_{L_t} S} \right) \quad (11)$$

Now, equation (9) becomes:

$$S_g = \frac{W}{\rho \bar{v}^2 \left( \frac{T_0}{W} - \mu \sqrt{C_{L_t} + \Delta C_{L_s}} \right) - R} \quad (12)$$

Since  $C_{L_t}$  has previously been taken to be the FC augmented wing lift coefficient at the take-off altitude, the question arises whether the increment of lift due to slipstream should be computed using the basic wing lift coefficient or the lift coefficient after augmentation by FC.

Equation (10) may be checked using the momentum theory, to determine whether the effect of velocity increment in the slipstream and the effect of FC are actually interdependent. Assume that the effect of FC is simply to increase the downwash angle, and that the effect of the propeller is simply to increase the air velocity in the slipstream without affecting the downwash angle. Then, for no slipstream present, the resultant force on the airplane wing, assumed nearly equal to the lift, is

$$R_0 = \frac{\rho v_b^2}{4} v_0^2 C \quad (13)$$

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The resultant force with slipstream present is approximately (Fig. 1)

$$\begin{aligned}
 F &= F_0 + \Delta F = \frac{\rho \pi b^2}{4} v_0^2 \epsilon - \frac{\rho \pi D^2}{4} v_0^2 \epsilon + \frac{\rho \pi D^2}{4} (v_0 + \Delta v)^2 \epsilon \\
 &= \frac{\rho \pi b^2}{4} \epsilon v_0^2 + \frac{\rho \pi \epsilon D^2}{4} (2v_0 \Delta v + \Delta v^2) \\
 \text{or } \Delta F &= \frac{\rho \pi \epsilon D^2}{4} (2v_0 \Delta v + \Delta v^2) \tag{14}
 \end{aligned}$$

with one propeller of diameter D.

In terms of lift coefficient increment

$$\Delta c_{Ls} \frac{\rho}{2} s v_0^2 = \frac{\rho \pi \epsilon D^2}{4} (2v_0 \Delta v + \Delta v^2)$$

or

$$\Delta c_{Ls} = \frac{\pi \epsilon AR \epsilon}{2} \frac{D^2}{b^2} \frac{(2v_0 \Delta v + \Delta v^2)}{v_0^2}$$

But from (13),

$$c_{L0} = \frac{\pi}{2} AR \epsilon$$

so

$$\Delta c_{Ls} = c_{L0} \frac{D^2}{b^2} \frac{(2v_0 \Delta v + \Delta v^2)}{v_0^2} \tag{15}$$

which is similar in form to equation (10). Therefore the value of  $c_L$  to be used in equation (10) is that augmented by BLC. Unpublished results of wind tunnel tests conducted at the University of Wichita confirm the above.

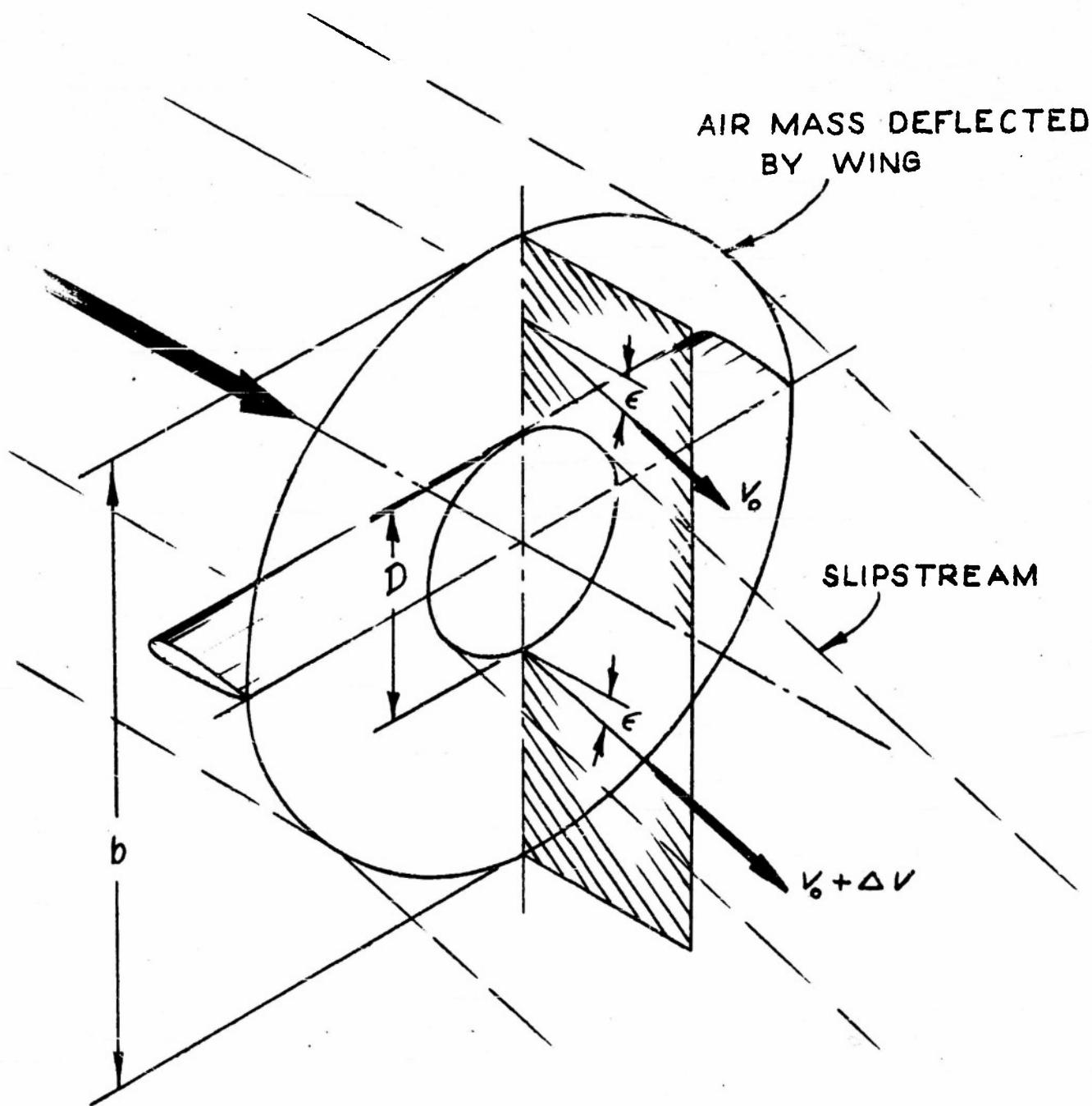


FIG. 1 - NOTATION FOR MOMENTUM APPROXIMATION OF PROP EFFECT

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